

ON THE GREATEST NATURAL NUMBER OF DEFINITIONAL OR INFORMATION COMPLEXITY $\leq N$

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The growth of this number $a(n)$ as a function of n serves to measure a number of very general phenomena.

For example, consider the time $t(n)$ it takes that program with not more than n bits to halt that takes the longest time to halt. This grows with n approximately in the same way as $a(n)$. More exactly, for all n , $a(n) \leq t(n + c)$ and $t(n) \leq a(n + c')$.

Consider those programs that halt and whose output is the set $S(n)$ of all binary strings of complexity not greater than n . Any program that halts and whose output includes $S(n)$, must either have more than $a(n - c)$ bits, or must take a time to halt exceeding $a(n - c)$. Both extremes are possible: few bits of program and very long running time, or vice versa. Thus those programs with about n bits which halt and whose output set is precisely $S(n)$ are among the programs of length n

that take most time to halt.

Or consider a program that outputs the r.e. but not recursive set of all programs that halt. The time it takes this program to output all programs of length not greater than n that halt, grows with n approximately like $a(n)$.

Or consider the set P having a binary string iff the string's information or definitional complexity is less than its length. P is "simple", that is, P is r.e. and its complement with respect to the set of all binary strings is infinite and contains no infinite r.e. subset. In fact, P is closely related to Post's original construction of a simple set. The time that it takes a program that outputs P to output all P 's elements of length not greater than n , grows with n approximately like $a(n)$.

Each of these results can be interpreted as the precise measure of a limitation of formal systems. For example, a formal system can be sufficiently powerful for it to be possible to prove within it that each program that halts in fact does so. Suppose that it is only possible to prove that a program halts if this is true. Then the maximum length of the proofs needed to establish that each program of length not greater than n that halts in fact does so, grows with n in approximately the same manner as $a(n)$.

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