ON THE LENGTH OF PROGRAMS FOR COMPUTING FINITE BINARY SEQUENCES BY BOUNDED-TRANSFER TURING MACHINES II

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Abstract 631–6. G. J. Chaitin, 819 Madison Avenue, New York, New York 10021. *On the length of programs for computing finite binary sequences by bounded-transfer Turing machines. II.*

Refer to Abstract 66T–26, these *Notices 13* (1966), 133. There it is proposed that elements of C_n may be considered patternless or random. This is applied; some properties of the L function are derived by using what may be termed the simple normality (Borel) of these binary sequences.

Note that

$$\textbf{(4)}\ L(S * S') \leq L(S) + L(S'),$$

where S and S' are finite binary sequences and $*$ is the concatenation operation. Hence

$$L(C_{n+m}) \leq L(C_n) + L(C_m).$$

With (1) this subadditivity property yields

$$\textbf{(5)}\ an \leq L(C_n)$$

(actually, subadditivity is used in the proof of (1)). Also,

(6) for any natural number k, if an element of C_n is partitioned into successive subsequences of length k, then each of the 2^k possible subsequences will occur $\sim 2^{-k}(n/k)$ times.

(6) follows from (1) and a generalization of (3). (4), (5) and (6) give immediately

$$\textbf{(7)}\ an \leq 2^{-n} \sum L(S),$$

where the summation is over binary sequences S of length n. Denote the binary sequence of length n consisting entirely of zeros by 0^n. As $L(0^n) = O(\log n)$, for n sufficiently large

$$L(C_n) > 2^{-n} \sum L(S) \geq an,$$

or

$$\textbf{(8)}\ an < L(C_n).$$

For each k it follows from (4) and (6) that for s sufficiently large

$$L(C_s) = L(S) = L(S' * 0^k * S''),$$

where

$$S' * 0^k * S'' = S \in C_s,$$

so that

$$L(C_s) \leq L(C_n) + L(C_m) + L(0^k) \quad (n + m = s - k).$$

This last inequality yields

$$\textbf{(9)}\ (L(C_n) - an) \text{ is unbounded.}$$

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