

INFORMATION- THEORETIC ASPECTS OF THE TURING DEGREES

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Abstract 72T-E77. GREGORY J. CHAITIN, Mario Bravo 249, Buenos Aires, Argentina. *Information-theoretic aspects of the Turing degrees*. Preliminary report.

Use is made of the concept of the relative complexity of a finite binary string in one or more infinite binary strings. An infinite binary string is recursive in another iff $\exists c \forall n$ the relative complexity of its initial segment of length n is less than $c + \log_2 n$. With positive probability, an infinite binary string has the property that the complexity of its initial segment of length n relative to the rest of the string is asymptotic to n . One such string R recursive in \emptyset' is defined. This infinite string R is separated into \aleph_0 independent infinite strings, i.e. the complexity of the initial segment of length n of any of these strings relative to all the rest of these strings is asymptotic to n . By joining these independent infinite strings one obtains Turing degrees greater than $\mathbf{0}$ and less than $\mathbf{0}'$ with any denumerable partial order. From the fact that R is recursive in \emptyset' it follows that there is a recursive predicate P such that asymptotically n bits of axioms are needed to determine which of the following n propositions are true and which are

false: $\exists x \forall y P(x, y, a) (a < n)$.
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