

ON THE DIFFICULTY OF GENERATING ALL BINARY STRINGS OF COMPLEXITY LESS THAN N

AMS Notices 19 (1972), p. A-764

Abstract 72T-E101. GREGORY J. CHAITIN, Mario Bravo 249, Buenos Aires, Argentina. *On the difficulty of generating all binary strings of complexity less than n .* Preliminary report.

Complexity is taken in the information-theoretic sense, i.e. the complexity of something is the number of bits in the shortest program for computing it on the standard universal computer.

Let

$$\alpha(n) = \min \max(\text{the length of } P \text{ in bits, the time it takes } P \text{ to halt}),$$

where the minimum is taken over all programs P whose output is the set of all binary strings of complexity less than n .

Let

$$\beta(n) = \max f(n),$$

where the maximum is taken over all number-theoretic functions f of complexity less than n .

Let

$$\gamma(n) = \sum(\text{the length of } S),$$

where the sum is taken over all binary strings S of complexity less than n .

Take $f \approx g$ to mean that there are c and c' such that for all n , $f(n) \leq g(n + c)$ and $g(n) \leq f(n + c')$.

Theorem. $\alpha \approx \beta \approx \gamma$.

(Received June 19, 1972.)