Name:

This exam is open book, open notes, but there can be no sharing of any material. You can use the Internet, but only as a library. If you are not sure if something is allowed, please ask. If you use any material other than the textbook and your own prior work, you must prominently indicate the source in your answers.

Questions marked with ⋆ are optional and may be answered for extra credit. There are 7 questions (including one ⋆ question) on 11 pages. You have *60 minutes* to earn *60 non-⋆ points*. You may wish to use this correspondence to plan your time.

Unless otherwise indicated, use the textbook's definitions for all terms, such as trees, balancing conditions, and red-black tree insertions and rotations.

1. (1 pt.) Write your name in the space provided above.

2. (14 pts.) For each statement below, indicate whether it is true or false by writing *True* or *False* in the blank next to it. *Justify your answer briefly.*

    (a) A red-black tree may be described as a *2,3,4-tree*. _____

    (b) An AA-tree may be described as a *2,3,4-tree*. _____

    (c) In a B-tree (as described in the textbook and in class) of order M, each interior node has between $\lceil M/2 \rceil$ and M children. _____

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(d) External sorting using k-way merges requires at least $2k$ tapes (for all $k > 1$). _____

(e) Splay trees are balanced binary search trees in which every node that is accessed is rotated to root. _____

(f) The find operation in a splay tree containing n nodes requires $O(\log n)$ time in the worst case. _____

(g) It is not possible for a single splay operation to use more than one *zig* step. _____

3. (10 pts.) Fill in the blank entries in the following table, indicating the number of runs on each of the five tapes used in a polyphase merge-sort of order 4. Row n of the table summarizes the distribution of runs on the tapes immediately following the nth merge. We number rows from 0 and the 0th row summarizes the initial distribution of runs (before any merges).

merge	number of runs on each tape				
	tape 1	tape 2	tape 3	tape 4	tape 5
0	10	9	5	6	0
1					
2					
3					
4					
5					

4. (15 pts.) Starting with the empty AA-tree, indicate how the tree changes as the following keys are inserted, in this order: 50, 99, 98, 31, 29, 46, 31, 41, 59, 42 For each insertion, indicate clearly (1) the state of the tree immediately following the insertion (before any balancing operations) as well as (2) the state of the tree after all balancing operations for that step have completed. When multiple balancing operations are used, indicate each step separately, identifying the operations used.

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5. (10 pts.) Indicate how the key 22 is inserted into the following binary heap. Show all steps, as in the textbook.[1]
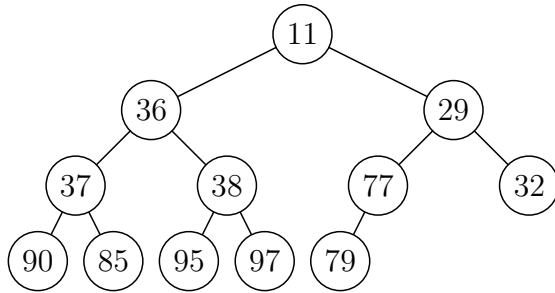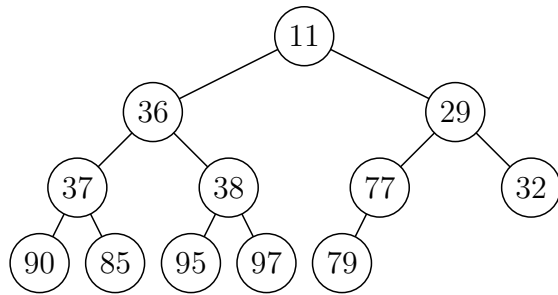

[1]Mark Allen Weiss, *Data Structures and Problem Solving Using Java*, 3rd edition (Addison-Wesley, 2006), Figures 21.7 and 21.8.

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6. (10 pts.) Indicate the heap that results from a *deleteMin* operation on the following binary heap. Show all steps, as in the textbook.[2]

[2] *Idem*, Figures 21.10–21.12.

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7. (10 pts.) ⋆ Using the method suggested by Reynolds's paper, determine the initial distribution of 82 runs on 6 tapes for a 5-way polyphase merge.

That is, indicate the number of runs initially written to each of the tapes, numbered 1 through 6. Show the intermediate steps used in arriving at the final distribution.

Then indicate the result of each merge step in tabular form, as in Question 3, until only one run remains.

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