

1. List the members of your group below:
  
  
  
  
  
  
  
  
  
  
2. A *subsequence* of sequence  $S$  is any sequence that can be obtained from  $S$  by deleting zero or more of its elements. For example,  $(1, 4, 9, 2)$  is a subsequence of  $S_1 = (3, 1, 4, 5, 9, 2)$ , but  $(1, 9, 4)$  is not. A subsequence  $S'$  of  $S$  is called a *k-subsequence* if each pair of adjacent elements in  $S'$  has  $k - 1$  intermediate elements in  $S$ . For example,  $(1, 5, 2)$  is a 2-subsequence of  $S_1$ , and  $(3, 5)$  is a 3-subsequence of  $S_1$ , but  $(1, 5, 9)$  is not a  $k$ -subsequence of  $S_1$  for any value of  $k$  (although it is a subsequence of  $S_1$ ). A  $k$ -subsequence with  $n$  elements is called *maximal* if there is no  $k$ -subsequence with  $n + 1$  elements. List all *maximal 5-subsequences* and *maximal 7-subsequences* of the following sequence:

50 40 60 70 65 75 62 63 41 42 51 52 53 54

3. We say a sequence is *k-sorted* if all of its *k*-subsequences are sorted. For each of the following, provide an example of a sequence with the indicated properties, or explain why no such sequence exists.
- (a) 7-sorted but not 5-sorted.
  - (b) 5-sorted but not 7-sorted.
  - (c) 6-sorted but not 3-sorted.
  - (d) 3-sorted but not 6-sorted.

4. Sort the following array in ascending order using *shellsort with increment sequence* (1, 5, 7).<sup>1</sup> Depict the state of the array after each *k*-sort, for *k* = 1, 5, 7 and highlight the moved elements at each stage.

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<sup>1</sup>Mark Allen Weiss, *Data Structures and Problem Solving Using Java*, 3rd edition (Addison-Wesley, 2006), §8.4.

5. Consider the process of sorting the array of Question 4 in ascending order using *merge-sort*.<sup>2</sup> Depict the recursive invocations of the `mergeSort` method using a tree in which nodes represent `mergeSort` invocations and are labeled with the indices of the sub-arrays sorted by them. Further, the parent of a node  $n$  is the node  $p$  corresponding to the `mergeSort` invocation (if any) from which  $n$ 's invocation is called.

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<sup>2</sup>*Idem*, §8.5.

6. Augment, or redraw, the tree of Question 5 by adding to each node's label the state of the sub-array corresponding to that node's invocation (1) immediately before the invocation and (2) immediately after the invocation.