1. List the members of your group below:

2. Let P_n, $n > 0$, denote the set of *permutations* of the elements $[n] = \{1, 2, \ldots, n\}$. That is, an element $p \in P_n$ is a sequence composed of the elements of $[n]$ in some order. For example, $(4, 1, 3, 2) \in P_4$.

   Enumerate P_n for $n = 1, 2, 3, 4$.

3. (Extra credit) Devise an efficient algorithm to generate all permutations of a given multiset of elements in lexicographic order.

4. For $p, q \in P_n$, we say p is a *swap neighbor* of q if swapping one pair of distinct elements in p yields q. List all swap neighbors of $(1, 2, 3)$, and of $(1, 2, 3, 4)$.

5. Is the swap-neighbor relation *symmetric*?

6. How many swap neighbors does a permutation $p \in P_n$ have? Justify your answer.

7. Define a *swap graph of n-permutations* as an undirected graph $G_s(n)$ whose vertices are permutations of P_n and whose edges connect pairs of swap-neighboring permutations. Depict $G_s(n)$ for $n = 0, 1, 2, 3, 4$.

8. For a permutation $p \in P_n$ and $1 \leq i \leq n$, we adopt the notation $p[i]$ for the i'th element of p and $p^{-1}[j]$ for the index of the element j (so that $p[p^{-1}[j]] = j$). Let $1 \leq x \leq n$ be an element in permutation $p \in P_n$ such that $p^{-1}[x] \neq x$. Define the *rotate home* operation $S(p, x)$ as one that

   - shifts elements $p[x], \ldots, p[p^{-1}[x] - 1]$ to the left if $x > p^{-1}[x]$, and $p[p^{-1}[x] + 1], \ldots, p[x]$ to the right otherwise; and
   - sets $p[x] \leftarrow x$.

   Evaluate $S((1, 3, 5, 4, 6, 2), 2)$ and $S((5, 1, 4, 2, 3, 6), 5)$.

9. Define a *rotate-home graph of n-permutations* as a directed graph $G_r(n)$ that has a vertex for each permutation of P_n and an edge from p to q if there is an element x such that $S(p, x) = q$. Depict $G_r(n)$ for $n = 0, 1, 2, 3, 4$.

10. How many *undirected cycles* and *directed cycles* does $G_r(n)$ contain, for $n = 0, 1, 2, 3, 4$?

11. Define a *rotate leftmost home* operation on a permutation $p \in P_n$, $p \neq (1, 2, \ldots, n)$ as the operation $S_l(p) = S(p, x)$ where x is the leftmost element in p such that $p[x] \neq x$.

    Depict the sequence of permutations obtained by repeated application of the rotate leftmost home operation to the sequence $(2, 3, 4, \ldots, n, 1)$ for $n = 2, 3, 4$.

12. Does the procedure outlined in Question 11 always terminate? Explain your answer.