Name: ____

1. (1 pt.) Write your name in the space provided above.

2. (6 pts.) Determine the binary search tree produced when the following operations are applied, *in the order presented*, to an initially empty tree. *Depict the state of the tree after each operation*, labeling your trees appropriately. *Use the textbook's specific methods in all cases.*

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insert(5), insert(17), insert(11), insert(13), insert(27),
insert(23), insert(2), remove(17), insert(3), remove(11),
insert(17), remove(2).
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[additional space for answering the earlier question]

3. (6 pts.) Let T_k denote the tree resulting from the k'th operation of Question 2. Fill in the blanks in the following table noting the *internal path length* and *external path length*.

k:	01	03	05	07	09	11
IPL(T_k):						
EPL(T_k):						

4. (6 pts.) We represent the empty binary tree by $\emptyset$ and a nonempty binary tree with root n, left subtree l, and right subtree r by the triple (n, l, r). Consider the following function f_1 on binary trees:

$$f_1(t) = \begin{cases} (n, \emptyset, \emptyset) & \text{if } t = (n, \emptyset, \emptyset) \\ (n, \emptyset, f_1(l)) & \text{if } t = (n, l, \emptyset) \text{ and } l \neq \emptyset \\ (n, f_1(r), \emptyset) & \text{if } t = (n, \emptyset, r) \text{ and } r \neq \emptyset \\ (n, f_1(l), f_1(r)) & \text{if } t = (n, l, r) \text{ and } l, r \neq \emptyset \\ \emptyset & \text{otherwise} \end{cases}$$

Depict, using the usual graphical conventions, the binary tree $f_1(T_9)$ where T_9 is defined in Question 3:

5. (6 pts.) We use the notation $f^k(t)$ (with $k > 0$) to denote k nested applications of the function f, that is, $f(f(f(\ldots f(t))))$, where there are k instances of f in the expression. Using the definition of f_1 from Question 4, depict, using the usual graphical conventions, the binary trees $f_1^{20}(T_{11})$ and $f_1^{21}(T_{11})$ where T_{11} is defined in Question 3. *Explain your answers.* (There is no credit for answers without proper explanations.)