This exercise reviews some concepts related to permutations and graphs and explores an interesting application of Eulerian graphs to permutations.

1. List the members of your group below. Underline your name.

2. Define *sorting* and *comparison sorting*.

3. Name a well-known comparison-sorting algorithm and another sorting algorithm that is not a comparison-sort.

4. Define a *permutation* of a collection of objects. List all permutations of the collection $\{1, 1, 3, 5, 5\}$.

5. For an integer $n > 1$, let $V_n$ be the set of $(n-2)$-character strings $\{x_1 x_2 \ldots x_{n-2} \mid x_i \in \{1, 2, \ldots, n\}$ and $x_i \neq x_j$ unless $i = j\}$.
   (a) List $V_n$ for $n = 2, 3, 4$.
   (b) What is the cardinality of $V_n$, as a function of $n$?
   (c) Provide an alternate, equivalent (perhaps simpler) definition of $V_n$.

6. For an integer $n > 1$, define a digraph $Q_n = (V_n, E_n)$ where the set of vertices $V_n$ defined in Question 5 and the set of edges $E_n = \{(u, v) \mid u, v \in V_n \text{ with } u = x_1x_2x_3 \cdots x_{n-2},\ v = x_2x_3 \cdots x_{n-2}x_{n-1},\ \text{where } x_i \neq x_j \text{ for } i \neq j\}$.
   (a) Depict $Q_n$ for $n = 2, 3, 4$.
   (b) What is the cardinality of $E_n$, as a function of $n$?
   (c) Is there anything notable about the degrees of vertices in $Q_n$?
   (d) Provide an alternate, equivalent (perhaps simpler) definition of $Q_n$.

7. Do the graphs $Q_2$, $Q_3$, and $Q_4$ of Question 6 have Eulerian paths? For each graph, exhibit an Eulerian path or explain why no such path exists.

   Recall that an Eulerian path in a digraph is a directed path that traverses each edge exactly once. A digraph with such a path is called Eulerian.

8. Prove or disprove: The graphs $Q_n$ of Question 6 are Eulerian for all $n > 1$.