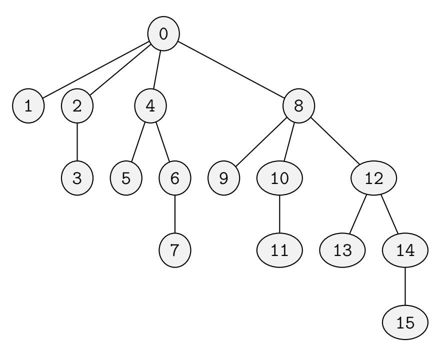
Today: Synthesis, minimum spanning trees, union-find data structures; §§ 24.*.
Next class: External sorting, polyphase merging, synthesis; § 21.6, Reynolds's paper.[1]
Reminders: Use the newsgroup. Term projects exhibition with posters next week.

1. Write your group identifier (e.g., C3) and its members' names. Underline your name.

2. (a) Provide an illustrative example of an equivalence relation R by specifying the base set S of elements and the pairs $x, y \in S$ for which xRy is true. Do not reuse the examples in the textbook.
   (b) Repeat for relations R_1, R_2, and R_3 which are not equivalence relations because they violate the requirements of reflexivity, symmetry, and transitivity, respectively (with each R_i satisfying the other two requirements).

3. Starting with each of the items $0, 1, 2, \ldots, 15$ in a singleton set by itself, provide an explicit sequence (as short as possible) of operations that yields the following tree (similar to Figure 24.17 in the textbook) when using the union-by-size smart-union algorithm (without path compression). Depict the forest before and after the *last two* operations.



[1]Samuel W. Reynolds, "A Generalized Polyphase Merge Algorithm," *Communications of the ACM* 4/8 (1961).

[additional space for answering the earlier question]

4. Depict the tree resulting from *find*(7) applied to the tree of Question 3 when *path compression* is in use. How many nodes get their parents changed?

5. Consider the effect of *path compression* on the structure of Question 3. Suppose we are permitted to augment the sequence of operations of Question 3 by introducing two *find* operations of our choice into the sequence, at positions also of our choice. If our goal is to minimize the height of the resulting structure, what are the best choices for the find operations and their positions? Explain.

6. $\star$ Sketch $t(5)$. How many parentheses does $t(k)$ have? Relate to union-find structures.
$t(0) = ()$
$t(k) = (t(k-1), t(k-2), t(k-3), \ldots, t(0)), \quad k > 0$