

Name: \_\_\_\_\_

1. (1 pt.)

- **Read all material carefully.**
- *If in doubt whether something is allowed, ask, don't assume.*
- You may refer to your books, papers, and notes during this test.
- E-books may be used *subject to the restrictions* noted in class.
- No computer or network access of any kind is allowed (or needed).
- Write, and draw, carefully. Ambiguous or cryptic answers receive zero credit.
- Use class and textbook conventions for notation, algorithmic options, etc.

Write your name in the space provided above.

2. (14 pts.) Trace the execution of the FIND-MAX-CROSSING-SUBARRAY algorithm on the array **A** depicted below, with the arguments **low**, **mid**, and **high** equal to 1, 5, and 10, respectively.

i:	1	2	3	4	5	6	7	8	9	10
A[i]:	88	19	9	-66	-2	116	-56	-12	87	101

List the values of *sum* and *left-sum* after each iteration of the first for-loop of the algorithm. Similarly, list the values of *sum* and *right-sum* after each iteration of the second for-loop.

3. (15 pts.) Depict the recursion tree that outlines the recursive calls made by the FIND-MAXIMUM-SUBARRAY algorithm when invoked on the array of Question 2 (repeated below), with `low` and `high` equal to 1 and 10, respectively. The nodes of the tree should be labeled with the function invoked (FIND-MAXIMUM-SUBARRAY or FIND-MAX-CROSSING-SUBARRAY and the edges should connect each function's node to the node of its invoker.

i:	1	2	3	4	5	6	7	8	9	10
A[i]:	88	19	9	-66	-2	116	-56	-12	87	101

4. (10 pts.) List all *derangements* of the sequence 1, 2, 3, 4.

5. (10 pts.) Let  $!n$  denote the number of derangements of a sequence of  $n$  distinct items. Prove or disprove:  $!n = (n - 1)(!(n - 1) + !(n - 2))$  for  $n > 1$ .

6. (10 pts.) Prove or disprove: The following algorithm generates a *uniform random permutation* of an array  $v$  when invoked as  $\text{foo}(v)$ . (The function  $\text{Random}(a,b)$  is as defined in the textbook.)

```
foo(v) {
  n = v.length
  bar(v, n, 1, n)
}

bar(v, n, lo, hi) {
  if lo < hi then {
    if Random(0,1) < 1 then {
      swap v[lo] with v[Random(1,n)]
      foo(v, lo + 1, hi)
    }
    else {
      swap v[hi] with v[Random(1,n)]
      foo(v, lo, hi - 1)
    }
  }
}
```

[additional space for answering the earlier question]