Name: ______________________________

1. (1 pt.)

   - **Read all material carefully.**
   - You may refer to your books, papers, and notes during this test.
   - No computer or network access of any kind is allowed (or needed).
   - Write, and draw, carefully. Ambiguous or cryptic answers receive zero credit.
   - Use textbook and classroom conventions for notation, algorithmic options, etc.
   - Ask for clarifications on the above if needed.
   - The question marked with a ⋆ is
     - required for COS 550, but
     - optional (extra credit, graded more strictly than non-⋆) for COS 451.
   - *COS 550 students* (only) get *10 extra minutes.*

   Write your name in the space provided above.

WAIT UNTIL INSTRUCTED TO CONTINUE TO REMAINING QUESTIONS.

2. (19 pts.) Prove or disprove (separately):

   (a) PTIME is closed under complementation.

   (b) NPSPACE is closed under union.

   (c) The set of all languages (over a finite alphabet) is countable.

[additional space for answering the earlier question]

3. (20 pts.) Use the textbook's method to convert the following regular expression into an equivalent NFA.

   $a(b \cup cd)^*a(a \cup bb)$

[additional space for answering the earlier question]

$a(b \cup cd)^*a(a \cup bb)$

4. (20 pts.) Convert the following grammar to Chomsky normal form. Upper-case letters denote variables and lower-case letters denote terminals. *Show enough intermediate results and include brief explanations* to make it clear that the method described in the textbook is being followed.

$$
\begin{aligned}
S &\rightarrow AaaB \mid aBbA \mid Sa \mid bSb \\
A &\rightarrow abb \mid aAB \mid \epsilon \\
B &\rightarrow b \mid SSb \mid \epsilon
\end{aligned}
$$

[additional space for answering the earlier question]

5. (20 pts.) Using the tabular representation used in class, depict the operation of the CYK algorithm on the input string `aabaabaaaa` and the final (Chomsky normal form) grammar of Question 4.

6. (20 pts.)

- Reduce the following instance of TQBF to an instance of GG (Generalized Geography) using the textbook's method.
- Determine the solution to either the TQBF or GG instance (your choice).
- Use the above solution to one instance to determine the solution to the other instance. Briefly explain your answer.

$$\exists x \forall y \exists z \forall w [(w \vee \neg x \vee z) \wedge (\neg w \vee \neg y \vee z) \wedge (x \vee y \vee \neg z) \wedge (\neg x \vee y \vee \neg z) \wedge (\neg w \vee \neg x \vee z)]$$

[additional space for answering the earlier question]

$$\exists x \forall y \exists z \forall w [(w \vee \neg x \vee z) \wedge (\neg w \vee \neg y \vee z) \wedge (x \vee y \vee \neg z) \wedge (\neg x \vee y \vee \neg z) \wedge (\neg w \vee \neg x \vee z)]$$

7. (15 $\star$ pts.) Prove or disprove the following (separately):

   (a) If L is a language then: $L \in \mathrm{NP}$ iff $L^* \in \mathrm{NP}$ (where L^* is the language of strings composed of the concatenation of zero or more strings from L).

   (b) The following language L is undecidable:
   $L = \{\langle G \rangle \mid G \text{ is a CFG over alphabet } \Sigma \text{ and } \exists x \in \Sigma, \exists k \in \mathbb{N} : x^k \in L(G)\}$

[additional space for answering the earlier question]