Name: _____

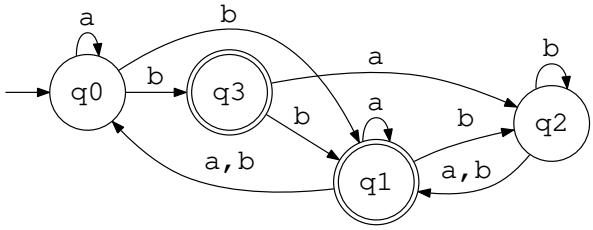
1. (1 pt.)
   - **Read all material carefully.**
   - You may refer to your books, papers, and notes during this test.
   - No computer or network access of any kind is allowed (or needed).
   - Write, and draw, carefully. Ambiguous or cryptic answers receive zero credit.
   - Use textbook and classroom conventions for notation, algorithmic options, etc.
   - Ask for clarifications on the above if needed.
   - The question marked with a ⋆ is
     - required for COS 550, but
     - optional (extra credit, graded more strictly than non-⋆) for COS 451.
   - *COS 550 students* (only) get *10 extra minutes*.

   Write your name in the space provided above.

2. (19 pts.) Consider the language L_1 of binary strings in which the absolute value of the difference between the number of zeros and number of ones is a multiple of five. Is L_1 regular? If so, depict a FSA that recognizes the language, and prove that claim. Otherwise, use the pumping lemma to prove nonregularity.

[additional space for answering the earlier question]

3. (20 pts.) Generate a regular expression that is equivalent to the following finite-state automaton. *Show enough intermediate results and include brief explanations* to make it clear that the method described in the textbook is being followed.



[additional space for answering the earlier question]

[additional space for answering the earlier question]

4. (10 $\star$ pts.) Define the *k-interleaved language* of languages L_1 and L_2 to be the language

$$I_k(L_1, L_2) = \{x_1 y_1 x_2 y_2 \cdots x_k y_k \quad | \quad x_1 x_2 \cdots x_k \in L_1 \wedge \\ y_1 y_2 \cdots y_k \in L_2 \wedge \\ \forall i \in [1, k] : x_i, y_i \in \Sigma^*\}$$

and define the *interleaved language* (no k) to be the language

$$I_*(L_1, L_2) = \bigcup_{k \geq 0} I_k(L_1, L_2)$$

Prove or disprove each of the following statements separately.

(a) If L_1 and L_2 are regular then $I_k(L_1, L_2)$ is regular.
(b) If L_1 and L_2 are regular then $I_*(L_1, L_2)$ is regular.
(c) If L_1 and L_2 are regular then $I_*(L_1, L_2)$ is context-free.

[additional space for answering the earlier question]

[additional space for answering the earlier question]