and we immediately obtain conditions on the coefficients for the quartic to be the square of a quadratic.

Finally some generalisations, suppose $P(x)$ is a polynomial of degree $d = pq$ where $p, q$ are integers $\neq 1$. We can investigate the possibility that $P(x)$ is the $p$th power of a polynomial of degree $q$ or, alternatively, that it is the $q$th power of a polynomial of degree $p$. For example a certain sextic might be the cube of a quadratic or the square of a cubic. In the first case

$$P(x) = Q(x)^3,$$

where

$$Q = \frac{p^{(iv)}}{72} - \left(\frac{p^{(v)}}{360}\right)^2.$$

In the second case

$$P(x) = R(x)^2,$$

where

$$R = \frac{1}{12} \left\{ p^{iv} - \frac{P^{(iv)} P^{(v)}}{960} + \frac{3}{4} \left(\frac{P^{(v)}}{120}\right)^3 \right\}.$$

Clearly once we get beyond the sextic the formulae are going to get more complicated. The formula for expressing a polynomial of degree $2p$ as the $p$th power of a quadratic is relatively easy to find—I leave it as an exercise for the reader—but I baulk at trying for the square root of the polynomial. Perhaps readers will have other ideas and experiences of how to overcome software inadequacies and/or hardware failures.

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75.3 Making a Golden Rectangle by paper folding

The Golden Ratio $((1 + \sqrt{5})/2 \approx 1.618)$ is a number with many interesting mathematical properties. The purpose of this short note is to show how you can construct a Golden Rectangle using just a sheet of $8.5\times11$" paper without any other tools. A Golden Rectangle is a rectangle such that the ratio of one dimension over the other is the Golden Ratio.

One problem in creating a Golden Rectangle out of an $8.5\times11$" sheet of paper is that the ratio of length/width is only about 1.29. Step 1 of the construction makes the paper narrow enough so we can create a Golden Rectangle from it. If for your sheet of paper the ratio of the longest dimension to the shortest dimension is greater than the Golden Ratio, then you can begin with Step 2.
1. Place the paper before you so its long dimension is horizontal. Make a horizontal fold in the paper in the exact middle of the sheet by folding the sheet in half. Fold the paper along the crease several times until the paper will tear readily along the crease. The final result will be a long thin rectangle $ABCD$.

2. Fold corner $A$ so it rests on $DC$ as shown in Fig. 1. Let $E$ be the new point created on $AB$ where the paper is folded.

3. Using $E$ as a guide, create a vertical crease at point $E$. Let $F$ be the point where the crease touches $DC$. In Fig. 2 the dashed line $EF$ represents the crease, and the figure $AEFD$ is a perfect square.

4. Now fold the paper so that $AD$ perfectly coincides with $EF$. This creates a vertical crease in the middle of the square. Mark the ends of this crease with $G$ and $H$ as shown in Fig. 3.

5. Fold $HC$ so that it runs through $E$ and mark $HC$ with a little fold where it intersects $E$ as shown in Fig. 4. Call this point $J$.

6. Make a vertical crease at $J$ by folding the paper. Call the other end of the crease $I$. Fig. 5 shows all the vertical creases created so far.

7. Fold the paper along $IJ$ sufficiently many times to ensure that it rips smoothly along $IJ$, then rip it along $IJ$. The final rectangle, $AIID$ in Fig. 6, is a Golden Rectangle.
To see that $AIJD$ of Fig. 6 is a Golden Rectangle, study Fig. 4 and use $x$ to denote the distance $DH$. For an 8.5" x 11" piece of paper it is easy to see that $x = 2.125"$. In terms of $x$, $GE$ has length $x$ and $GH$ has length $2x$. Since $HGE$ is a right triangle, the Pythagorean Theorem shows that $HE = HJ = x\sqrt{5}$. Thus, the size of $DJ$ is $x(1 + \sqrt{5})$ and since the size of $AD$ is $2x$ the ratio $DJ/AD$ is the Golden Ratio.

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CONTENTS

From quadrature to integration
"... Assume the string is inextensible and elastic ..."
Ján van Maanen 1
Tom Roper 15
and Ron Hartley
M. C. Jones 23
Tony Gardiner 27
Nigel Walkey 32
and Gerald Goodall
Neil Bibby 40
Kenneth Ruthven 48

Wherefore "plug-and-chug"?

Advanced calculators and advanced-level mathematics
Richard B. Wilson 55
Doug French 62
Philip Maher 68
P. M. H. Wilson 72
Adam C. McBride 75
Roger Heath-Brown 79

Notes 75.1–75.6
The tale of the lop-sided parabola
Nick Lord 80
Polynomial square roots
Nigel Backhouse 84
Making a Golden Rectangle by paper folding
George Markowsky 85
On the evaluation of certain improper integrals
Robert M. Young 88
An ancient Egyptian approximation
W. W. Wilson 89
and G. L. Wilson
Nick Lord 90

A sixth form booklist for the 1990s

Problem corner 93
Correspondence 96
Reviews 103