# Notes On Induction
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# Simple Induction

- The domain is of the form i...∞.
- There is only one stopping value, i. *(Base Case)*
- Inheritance is always P(n+1) inheriting from P(n). *(Inductive Case)*

# 4 Steps to Wisdom
# Proof by Simple Induction

- **Step 1. Define the Problem.**
  - Clearly define the domain D, which must be of the form $D=s...\infty$
  - Clearly state what you are doing induction on. In other words, what values are in the domain D? For example, if you are doing induction in connection with a graph, you might say something like “This proof is by induction on n, the number of vertices of the graph.”
  - Give names to all formulas and predicates that you might use.

# 4 Steps to Wisdom
# Proof by Simple Induction

- **Step 2. Checking Base Case & Two Other Values.**
  - **If any of the Base Cases does not work, then your claim is wrong and you need to go back to Step 1 and fix everything**
- **Step 3. Prove for all n > s, that if P(n) is true, then P(n+1) is true.**
- **Step 4. Conclude the Proof.**

# Prove $(n^3+2n) \bmod 3 = 0$

- Let's see how to do this as a simple proof by induction.

# Step 1. Define the Problem

- The domain D is the set of Natural Numbers, which can be represented by $0...\infty$
- Thus, 0 is the base case.
- There is a function F(n) and a predicate P(n) defined as below
- $F(n) = n^3 + 2n$
- $P(n) = (0 = F(n) \bmod 3)$

# Step 2. Check Stopping Value and Two Other Values

- $F(0) = 0^3 + 2*0 = 0$
- P(0) = True *(Base Case)*
- $F(1) = 1^3 + 2*1 = 3$
- P(1) = True
- $F(2) = 2^3 + 2*2 = 12$
- P(2) = True

# Step 3. Inheritance

- We assume that P(n) is true.
- This means that F(n) mod 3 is 0.
- $F(n) = n^3 + 2n$
- $F(n+1) = (n+1)^3+2(n+1) = n^3 + 3n^2 + 5n + 3$
- How can you relate F(n+1) to F(n)?
- Clearly, $F(n+1) = F(n) + 3n^2 + 3n + 3$
- Do you notice anything?

# Step 3. Inheritance

- $F(n+1) = F(n) + 3(n^2 + n + 1)$
- Thus, $F(n+1) \bmod 3 = F(n) \bmod 3 + (3 \bmod 3)*((n^2 + n + 1) \bmod 3)$
- $= 0 + 0*x = 0$

# Step 4. Conclude the Proof

- Since Steps 1-3 have been verified, it follows that the proof is completed.
- It follows that P(n) is true for all n in D
- Thus, $(n^3+2n) \bmod 3 = 0$ for all natural numbers n.