A clash of symbols: well-turned letters; wizards of odds

GAMES

By Scot Morris

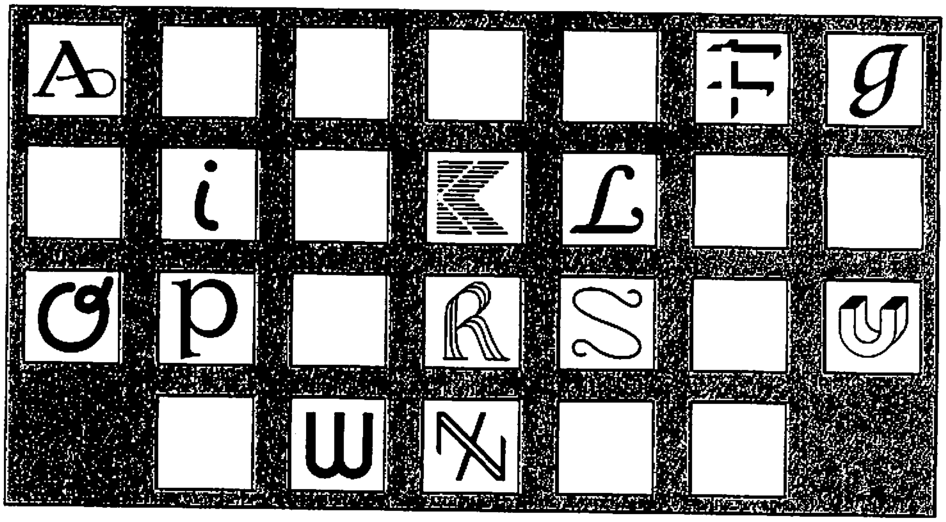
Scott Kim, whose visual wordplay called Inversions appeared for the first time in this column, has now come up with two new puzzles, called Halphabets, that are guaranteed to flip you out. The idea first dawned on him while he was designing a computer disk that allows nonartists to manipulate words and letterforms the way he does—so they can create their own Inversions.

In the Halphabet puzzles, you isolate each symbol (using the computer's mouse), rotate it 90° or 180° and/or reflect it, then copy the result in one of the empty squares. In the paper-and-ink versions printed here, the best way to see what these letters will look like reflected is to photocopy the page and hold it up to the light so that you can view the letter shapes from the back.

The two puzzles at right each start with half an alphabet—just 13 letters. The challenge: Copy each letter once and, by drawing it, either rotate or reflect it to fill one of the empty squares in the same "halphabet." In the end, each 26-letter alphabet is made up of two copies each of the original 13-letter forms.

For example, in the first quiz (Halphabet A) you can use the *W* to make either an *E* or an *M* but not both. Some symbols must be flipped to fit: A *p* can be rotated 180° to make a *d;* its mirror image can form a *b* or, if needed, a *q*.

Although the two Halphabets are separate puzzles,



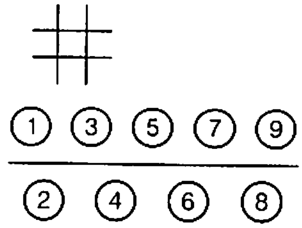
Kim made them complement each other. How did he do it? Answers appear on page 145.

The completed disk, *Letterforms & Illusion*, by Kim and Robin Fe Samelson, was recently published by W. H. Freeman, along with a rerelease of Kim's book *Inversions*. The Macintosh disk comes with ten original fonts, or letter styles, which lead to some 60 interactive puzzles. A user can write with any of the peculiar alphabets—including an "Escher" font and "illusion" font, an "extreme closeup" font, and a font in which each letter can tessellate a plane. (To order, send $39.95, plus $3 postage and handling, to Look Twice, Box 50697, Palo Alto, CA 94303.)

ODD MAN OUT?

The best games are simple enough to learn immediately and complex enough to keep you interested indefinitely. Tic-tac-toe, while simple, is too predictable: For every possible opening move by X there is a reply by O that will force a tie. The odd-even game, set up on a tic-tac-toe grid, may be easily learned, but you can play hundreds of times without discovering sure-fire strategies for winning or forcing a tie.

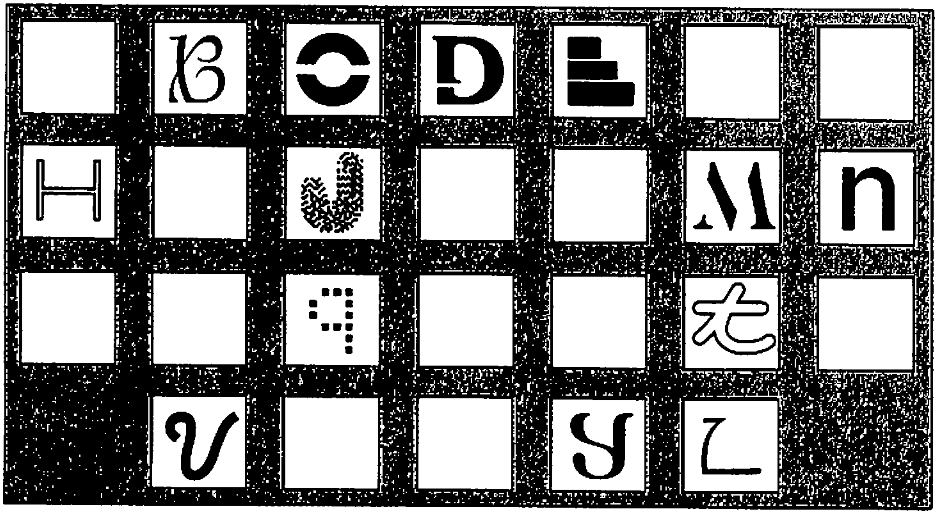
To play, draw a tic-tac-toe board, then write the digits *1* through *9*, separated into rows of odd and even numbers, as below.



One player, "Odd," takes charge of the digits *1, 3, 5, 7,* and *9;* the other player, "Even," gets *2, 4, 6,* and *8*. Odd goes first. Players alternate inserting one number in any empty cell. Numbers may be used only once (players cross them off as they use them), and once the numbers are placed, both players have access to them within the grid. The object: Be the first player to complete a row of three numbers (horizontal, vertical, or diagonal) whose sum is 15. The loser of a game chooses whether to be Odd or Even in the next game.

Ronald L. Graham invented odd-even in 1956. Now adjunct director for research at AT&T Bell Labs, Graham is listed in the *Guinness Book of World Records* for the eponymous Graham's Number—the largest number ever used in a mathematical proof.

In just a few matches, you'll see the difficulty of determining a "best play." Does the perfectly played game end in a win for Odd, a win for Even, or a tie? "I did a lot of pencil calculations, tracking out various strategies," Graham told me.



"I finally convinced myself that one side barely had a win, but all my analysis was done by hand—it took hundreds of pages—so I was never completely sure."

GRAHAM CRACKER

Graham's game was finally cracked by a computer this year. George Markowsky, professor of computer science at the University of Maine, attacked it with two programming languages, Pascal and APL. Markowsky sent me his analysis, published here for the first time.

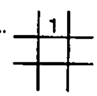
The first step in analyzing the game: Find the number of board configurations that are possible. For example, in tic-tac-toe, X can open in any of nine possible squares. Because one can rotate the grid, only three distinctly different openings really exist: to a corner square, to a midside square, or to the center square. Odd can place any of five numbers in any of the three distinct cells, for 15 possible openings. But every game has a numerical "mirror image"; games that start with *1* are symmetrical with games that start with 9. Opening with a *3* is equivalent to opening with a *7*.

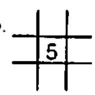
A game that has the first three moves *1–8–3* also has an exact reflection: *9–2–7*. Therefore Odd has only three distinctly different numbers (*1*, *3*, and *5*) and three distinctly different squares. This reduces the number of possible openings to nine.

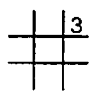
Markowsky's program pruned the 9,355,565 conceivable arrangements down to a more manageable (ha!) 514,816 distinct ones. A perfectly played game, it turns out, doesn't end in a draw. Odd *does* have a winning strategy. And knowing that, try to solve the following problems. (Answers appear on page 145.)

1. Of the three distinct opening squares (corner, midside, center), only one leads to a forced win for Odd. Which is it?

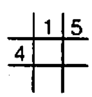
2. In the three following grids, determine which is the best opening for Odd and which is the worst.

a. 

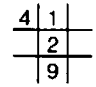
b. 

c. 

3. Below is game A after three moves. Even has 18 possible moves: the *2*, *6*, or *8* in any of the six open squares. Seventeen of these moves let Odd win on the next turn. What is Even's safest move?



4. In game B, Odd is to move. Place any of three numbers—*3*, *5*, or *7*—in any of five squares. Of the 15 possible moves, do the majority lead to (a) a win for Even, (b) a loss for Even, or (c) a tie?



There is no intuitive strategy to the game, so it remains challenging. Even though there is a best move for Odd in every position, it is almost impossible for any human to know what that move is without either trying it or looking up Markowsky's analysis.

There are other suggested variations on the game—for example, one that Markowsky analyzed using different colors and two ways to win. Play as before, but with the odd numbers on red counters, the even numbers on green. The object: Complete either a line of three that adds up to 15 or one that has three counters of the same color.

Other variations haven't been extensively analyzed. For example, no one knows how the game would differ if the desired sum were *13* or *14*. What if the numbers used were *2* through *10* and Even went first? And Graham's latest version: Put the numbers on cards, shuffle, and deal randomly. One player gets five numbers; the other gets four. The person with five opens. "The rule," Graham says, "leads to a whole range of unexplored games, with different strategies for each possible deal."

The level of complexity for odd-even is greater than any noughts-and-crosses player ever imagined.